Diffusion and mean free path

We've learnt that the dynamics of quasiparticles in a metal is described by the kinetic equation

$$\frac{\partial f(\vec{p},\vec{r})}{\partial t} + \vec{v}(\vec{p})\,\frac{\partial f(\vec{p},\vec{r})}{\partial \vec{r}} + e\vec{E}\,\frac{\partial f(\vec{p},\vec{r})}{\partial \vec{p}} = -\underbrace{\frac{f(\vec{p},\vec{r}) - f_0(\vec{p})}{\tau}}$$

Read: Abrikosov, chapter 3

Collision integral in the τ-approximation, may be more complicated

$f(\vec{p},\vec{r})$ – the density of quasiparticles in phase space

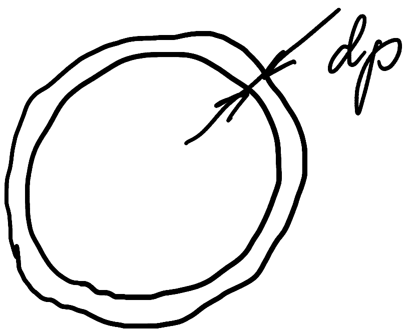
Using the kinetic equation, we derived the Drude formula for conductivity

$$\sigma = \frac{1}{3} e^2 v^2 \nu(\varepsilon_F)\,\tau$$

If we consider only elastic scattering and conductivity shows generated heat, how does it fit together?

[illegible] of the DoS

A brief rederivation of the DoS



$$\frac{4\pi p^2 dp}{(2\pi\hbar)^3} = \nu\, d\varepsilon \rightarrow \nu = \frac{p_F^2}{2\pi^2 \hbar^3 v_F}$$

(per spin)

In a system with spin,

$$\nu = \frac{p_F^2}{\pi^2 \hbar^3 v_F}$$

Then

$$\sigma = \frac{e^2 p_F^2 v_F \tau}{3\pi^2 \hbar^3} = \frac{e^2}{3\pi^2 \hbar} \frac{p_F}{\hbar} \frac{p_F l}{\hbar}$$

$$\sigma \sim \frac{e^2}{\hbar} (k_F l) \times \begin{cases} k_F, & 3D \\ 1, & 2D \\ k_F^{-1}, & 1D \end{cases}$$

$k_F l$ - Ioffe-Regel parameter

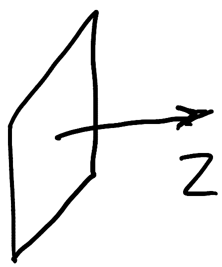
$k_F l \gtrsim 1$ in metals

$k_F \ell \gtrsim 1$ in metals

$\dfrac{e^2}{\pi\hbar}$ – conductance quantum

Relation between conductivity and diffusion

Let's assume there is some gradient of electron concentration along the z axis. Then there will be current along that axis



$$j(z) = \int n\, v \cos\theta \, \frac{d\Omega}{4\pi}$$

$$n = n(z - \ell \cos\theta)$$

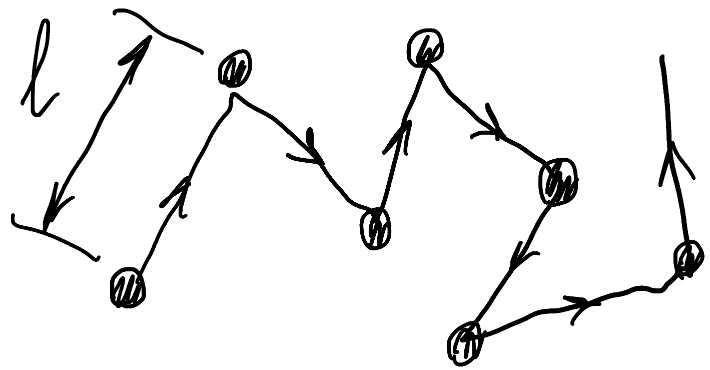
Mean free path

$$-\frac{dn}{dz}\,\ell v \int \cos^2\theta \, \frac{d\Omega}{4\pi} = -\frac{dn}{dz}\,\frac{\ell v}{3}$$

$D = \ell v \quad v^2 \tau$

diffusion coefficient

$$\boxed{D = \frac{l v}{3} = \frac{v^2 \tau}{3}}$$ – diffusion coefficient



Handwaving interpretation

$$\vec{R} = \vec{r}_1 + \vec{r}_2 + \ldots + \vec{r}_N$$ distance travelled after N collisions

$$\rightarrow R^2 = N l^2 \leftarrow$$

$$N \sim \frac{t v}{l}$$

$$\rightarrow R^2 \sim v l t \sim D t$$

$$\boxed{\sigma = e^2 D \nu(\varepsilon)}$$ – Einstein relation

$\sigma = e^2 D \nu(\varepsilon_F)$ – Einstein relation

The general solution of the diffusion equation

$$n(\vec{r},t) = \frac{1}{(4\pi D t)^{\frac{d}{2}}} \int n(\vec{R},0)\, e^{-\frac{(\vec{r}-\vec{R})^2}{4Dt}} d\vec{R}$$, i.e.

the solution of $\frac{\partial n}{\partial t} - D \Delta n = 0$

If all the particles are localised initially at $\vec{R}=0$, i.e.

$$n(\vec{R},0) = N\,\delta(\vec{R}),$$ then

$$n(\vec{r},t) = \frac{N}{(4\pi D t)^{\frac{d}{2}}} e^{-\frac{(\vec{r}-\vec{R})^2}{4Dt}}$$

Then $\langle R^2 \rangle = 2Dt$

Note: at finite temperature there may be collisions with phonons and with othe quasiparticles